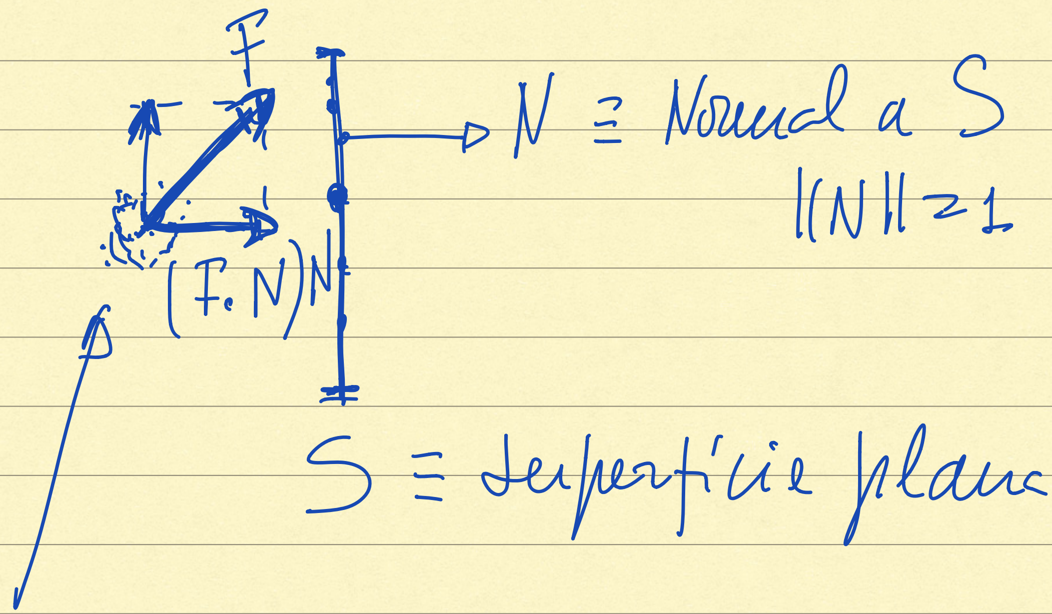
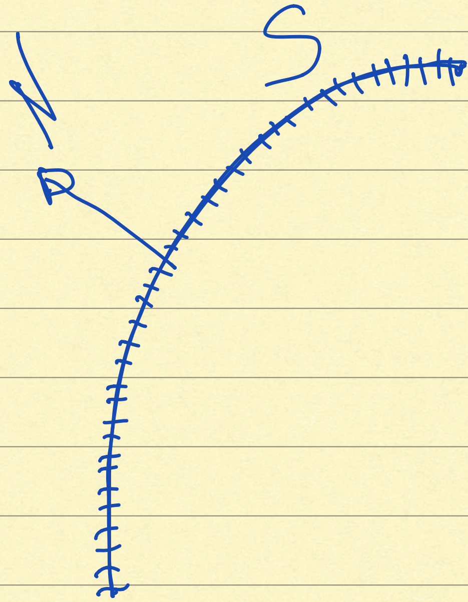
FLUXO de um campo vectorial através de uma superfície.



"partículas que passam através de S"

$$\underbrace{F\cdot N}\ \underbrace{\Delta S}_{\text{área}}$$

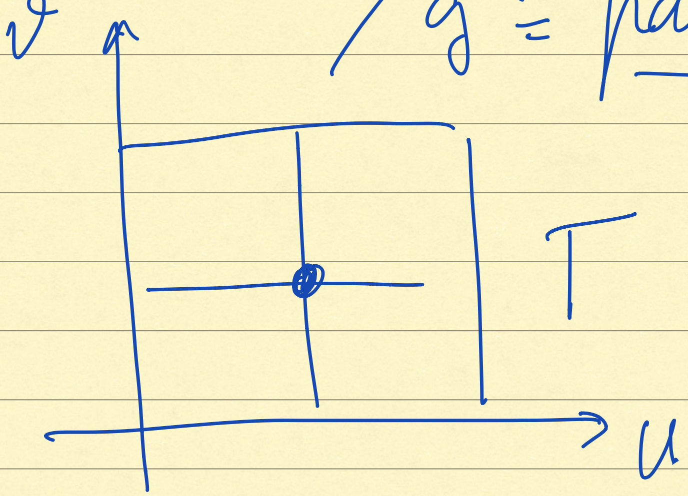


Definição: Fluxo de $F: \mathbb{R}^3 \to \mathbb{R}^3$ através de uma superfície $S \subset \mathbb{R}^3$ segundo a normal N:

$$\iint_S \underbrace{F \cdot N}\, dS$$

$$dS = \sqrt{\det Dg^T Dg}\; du\, dv$$

g = <u>parametrização</u>



$$\iint_S F\cdot N\, dS = \iint_T F(g(u,v))\cdot \underbrace{N(g(u,v))\sqrt{\det Dg^T Dg}}_{\text{complicado!}}\; du\, dv$$

$$S = \{ (x,y,z) \in \mathbb{R}^3 : H(x,y,z) = 0 \}$$

$H : \mathbb{R}^3 \to \mathbb{R}$, C^1.

$$\underbrace{N(x,y,z)}_{g(u,v)} = \frac{DH(x,y,z)}{\|DH(x,y,z)\|}$$

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Produto externo de vectores em $\mathbb{R}^3$

$A, B \in \mathbb{R}^3$, $\quad A \times B \in \mathbb{R}^3$

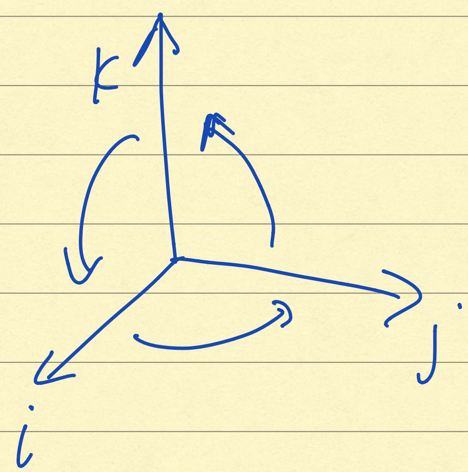
1) $(\alpha A + \beta B) \times C = \alpha A \times C + \beta B \times C$

$\forall \alpha, \beta \in \mathbb{R}$.

2) $A \times B = -B \times A$

3) $i = (1,0,0)$, $j = (0,1,0)$, $k = (0,0,1)$

$i \times j = k \; ; \quad j \times k = i \quad ; \; k \times i = j$



$$2) \Longrightarrow \quad A \times A = -A \times A = 0$$

$$A = A_1 i + A_2 j + A_3 k \qquad (A_1, A_2, A_3)$$

$$B = B_1 i + B_2 j + B_3 k \qquad (B_1, B_2, B_3)$$

$$A \times B = A_1 B_2 k - A_1 B_3 j +$$

$$- A_2 B_1 k + A_2 B_3 i +$$

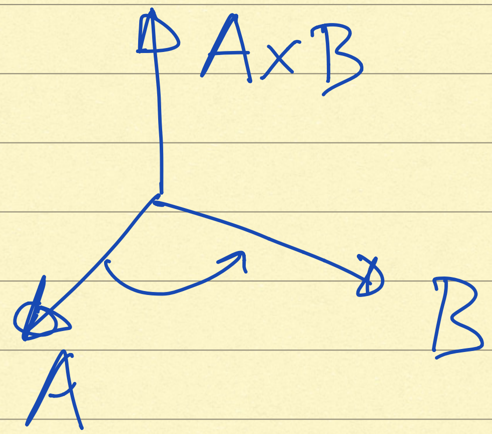
$$+ A_3 B_1 j - A_3 B_2 i$$

$$A\times B = \left(A_2B_3 - A_3B_2,\ A_3B_1 - A_1B_3,\ A_1B_2 - A_2B_1\right)$$

$$A\times B = {}'' \begin{vmatrix} i & j & k \\ A_1 & A_2 & A_3 \\ B_1 & B_2 & B_3 \end{vmatrix} '' \begin{array}{l} \\ \leftarrow A \\ \leftarrow B \end{array}$$

a) $A\times B\cdot A = 0$

$A\times B \cdot B = 0$



$$Dg(u,v) = \underbrace{\begin{bmatrix} \vdots & \vdots \\ D_u g & D_v g \\ \vdots & \vdots \end{bmatrix}_{3\times 2}}_{\text{Vectores tangentes.}}$$

$$\boxed{D_u g \times D_v g \equiv \text{vector normal!}}$$

b) $$\| D_u g \times D_v g \| = \sqrt{\det Dg^T Dg}$$

(exercicio)

$$\|N\| = 1 \qquad N = \frac{D_u g \times D_v g}{\|D_u g \times D_v g\|}$$

$$\iint_S F \cdot N \, dS = \iint_S F \cdot N \quad =$$

$$= \iint_T F(g(u,v)) \cdot \frac{D_u g \times D_v g}{\cancel{\|D_u g \times D_v g\|}} \cancel{\|D_u g \times D_v g\|} \, du \, dv$$

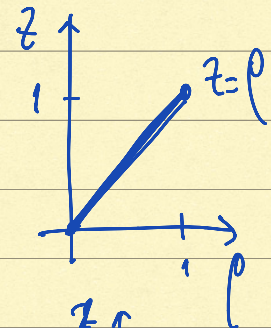
$$\boxed{\iint_S F \cdot N = \iint_T F(g(u,v)) \cdot (D_u g \times D_v g) \, du \, dv \qquad \text{"mais simples"} \; !}$$

Exemplo: S: $z=\sqrt{x^2+y^2}<1$

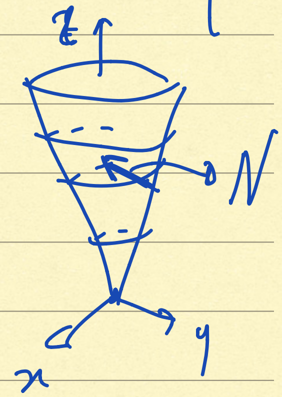
$\boxed{\text{Cone}}$

$F(x,y,z)=(-y,\ x,\ z)$

$$\iint_S F\cdot N \ ; \quad \boxed{N_z>0}$$



$z=\rho<1$; (ρ,θ,z)



$$g(\rho,\theta)=(\rho\cos\theta,\ \rho\,\mathrm{sen}\,\theta,\ \rho)$$

$$T\begin{cases} 0\leq\rho<1 \\ 0<\theta<2\pi \end{cases}$$

$$
\begin{array}{cccc}
 & i & j & k \\
D_\rho g = & (\cos\theta, & \operatorname{sen}\theta, & 1)
\end{array}
$$

$$D_\theta g = (-\rho \operatorname{sen}\theta,\ \rho\cos\theta,\ 0)$$

$$D_\rho g \times D_\theta g = (-\rho\cos\theta,\ -\rho\operatorname{sen}\theta,\ \rho)$$

$$\rho > 0$$

$$F(g(\rho,\theta)) = (\rho\operatorname{sen}\theta,\ \rho\cos\theta,\ \rho)$$

$$F(g(\rho,\theta)) \cdot D_\rho g \times D_\theta g = \rho^2$$

$$\iint_S F \cdot N = \int_0^{2\pi} \left(\int_0^1 \rho^2 \, d\rho \right) d\theta = \frac{2\pi}{3}$$